A RANDOM COLLISION MODEL ON GRAPHS(Combinatorial Aspects on the Analysis of Mathematical Models)

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グラフの上のランダムな衝突モデル

A RANDOM COLLISION MODEL ON GRAPHS

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We investigate a random collision model on graphs which is an idealized situation for interacting types. The types may represent species, genotypes, brands, factions or other classifications.

Consider a graph G of a finite nonempty set $V = V(G)$ of p points together with a specified set $X(G)$ of q unordered pairs of distinct points. A pair $x = \{u, v\}$ of points in X is called a line of G and x is said to join u and v. The points u and v are adjacent, u and v are incident with each other, as are v and x.

$W(u_1, u_2, \cdots, u_k)$ is the set of all pairs of distinct points $u_1, u_2, \cdots, u_k$. We consider a population of paricles of p types, $1, 2, \cdots, p$ which consist of $N_1(t), N_2(t), \cdots, N_p(t)$ respectively at time t. We write

$$\vec{N}(t) = (N_1(t), N_2(t), \cdots, N_p(t)) \tag{1}$$

It is assumed that initially $\vec{N}(0) = \vec{\alpha}$, that in each unit of time a pair of particles are chosen at random from the n particles , and that all possible pairs are equiprobably chosen. There exist adjacency relations between the types as defined by the graph G, with points $V(G) = \{1, 2, \cdots, p\}$ and adjacencies $X(G)$. Further it is assumed that a choice of a pair of particles of different types i and j, for which $(i, j) \in X(G)$, changes the pair of particles of the type i with probability $\frac{1}{2}$ and of the type j with probability $\frac{1}{2}$. A choice of a pair of particles of different types $(i, j) \in W(V) - X(G)$ does not make any effect, as well as a choice of a pair of the same type .

For each point $i \in V(G)$, consider variable x_i. $S_k(x_1, x_2, \cdots, x_p)$ is k-th order elementary symmetric function. $W(u_1, u_2, \cdots, u_k)$ is the set of all pairs of distinct points $u_1, u_2, \cdots, u_k$. Let $| S |$ be the number of the elements of the set S. Consider

$$T_{k,l}(x_1, x_2, \cdots, x(p)) = \sum_{|W(u_1,u_2,\cdots,u_k) \cap X(G)| = l} x_{u_1} x_{u_2} \cdots x_{u_k} \tag{2}$$

$\{N(t), t = 0, 1, 2, \cdots\}$ is a Markov chain with transition probabilities defined by

$$Pr(\vec{N}(t+1) = \vec{n}_{ij} \mid \vec{N}(t) = \vec{n}) = \frac{n_i n_j}{n(n-1)} \tag{3}$$

for $(i, j) \in X(G)$, where $\vec{n} = (n_1, n_2, \cdots, n_p)$, $n = \sum_{i=1}^{p} n_i$, $\vec{n_{ij}}$ is a vector with i-th component $n_i + 1$, j-th component $n_j - 1$, and all other components equal to those of $\vec{n}$. We have

$$Pr(N(t \vec{+} 1) = \vec{n} \mid \vec{N}(t) = \vec{n}) = \sum_{i=1}^{p} \frac{n_i^2}{n(n-1)} + \sum_{(i,j) \in W(V) - X(G)} \frac{2 n_i n_j}{n(n-1)} \tag{4}$$

Theorem. Let $Q(t)$ be the number of existing types at time t.

$$Pr(Q(t) \geq k) \geq \frac{T_{k,0}(\vec{N}(0))}{{}_M C_k (\frac{n}{M})^k} \tag{5}$$

where $M = max\{k \mid\mid W(u_1, u_2, \cdots, u_k) \cap X(G) \mid= 0\}$.

参考文献

[1] Itoh, Y. (1973). On a ruin problem with interaction, *Ann. Inst. Statist. Math.*, 25, 635-641

[2] Itoh, Y. (1979). Random collision models in oriented graphs, *J.Appl. Prob.*, 16, 36-44

[3] Itoh, Y. (1982). Random cllision models on graphs, *Pacific Area Statistical Conference*, 27-30

[4] Harary, F. (1969). Graph theory, Addison-Wesley Publishing Company, Inc., Massachusetts.